WRITTEN HOMEWORK #8, DUE MAR 5, 2010

(1) (Chapter 17.9, #23)
Verify that $\nabla \cdot \mathbf{E} = 0$ for the electric field $\mathbf{E}$ generated by a single particle of charge Q at the origin, given by Coulomb's Law

$$\mathbf{E} = \frac{Q}{|\mathbf{x}|^3}\mathbf{x},$$

where $\mathbf{x} = \langle x, y, z \rangle$.

(2) (Chapter 17.9, #24) Use the Divergence Theorem to evaluate

$$\iint_S (2x + 2y + z^2)\, dS$$

where S is the sphere $x^2 + y^2 + z^2 = 1$, with outward orientation.

(3) (Chapter 17.9, #4) Let $\mathbf{F} = \langle x, y, z \rangle$, and let S be the unit sphere $x^2 + y^2 + z^2 = 1$, with outward pointing orientation. Verify the Divergence Theorem by (a) calculating the surface integral of $\mathbf{F}$ across S directly, and (b) computing the integral of $\nabla \cdot \mathbf{F}$ over the interior of S, and checking that the two answers you get are the same.

(4) (Chapter 17.8, #5) Let $\mathbf{F} = \langle xyz, xy, x^2yz \rangle$. Let S be the the surface consisting of the top and four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, oriented outward. Evaluate the surface integral of $\nabla \times \mathbf{F}$ across S. (Hint: you probably want to use Stokes' Theorem.)

(5) (Chapter 17.8, #13) Let $\mathbf{F} = \langle y^2, x, z^2 \rangle$. Let S be the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upwards. Verify Stokes' Theorem for this $\mathbf{F}$ and S by calculating (a) the surface integral of $\nabla \times \mathbf{F}$ across S directly, and (b) the line integral of $\mathbf{F}$ along the boundary of S, and checking that your answers are the same.

(6) (Chapter 17.8, #16) Let C be a simple closed smooth curve that lies in the plane $x + y + z = 1$. Show that the line integral

$$\int_C z\, dx - 2x\, dy + 3y\, dz$$

depends only on the area of the region enclosed by C, and not on the shape of C or its location in the plane.